FACULDADE DE CIÊNCIAS E TECNOLOGIA
UNIVERSIDADE NOVA DE LISBOA

Mestrado em Matemática Atuarial

Syllabus

1st year/ 1st semester

Course	Scientific area	Period	Hours	Hours at school	ECTS	Obs
Life Contingencies I	Mathematics	Semester	168	TP: 56	6	Mandatory
Risk theory I	Mathematics	Semester	168	TP: 42	6	Mandatory
Stochastic Processes and Modelling	Mathematics	Semester	168	TP: 42	6	Mandatory
Non-Life Insurance Pricing	Mathematics	Semester	168	TP: 56	6	Mandatory
Machine Learning	Informatics	Semester	168	T: 28; PL: 28	6	Mandatory

1st year/ 2nd semester

Course	Scientific area	Period	Hours	Hours at school	ECTS	Obs
Advanced Life Contingencies	Mathematics	Semester	168	TP: 56	6	Mandatory
Risk Theory II	Mathematics	Semester	168	TP: 42	6	Mandatory
Claim Reserving	Mathematics	Semester	168	TP: 42	6	Mandatory
Actuarial Risk Management	Mathematics	Semester	84	TP: 28	3	Mandatory
Entrepreneurship	Transferable Skills	Semester	84	TP: 28	3	Mandatory
Unidade Curricular do Bloco Livre A	Any Scientific Area	Semester	168	Depends on the option	6	Optional

2nd year/ 1st semester

Course	Scientific area	Period	Hours	Hours at school	ECTS	Obs
Pension Funds and Social Security	Mathematics	Semester	168	TP: 56	6	Mandatory
Solvency Models	Mathematics	Semester	168	TP: 42	6	Mandatory
Health and Long Term Care Insurance	Mathematics	Semester	168	TP: 42	6	Mandatory
Investments Theory	Mathematics	Semester	168	TP: 42	6	Mandatory
Systems for Big Data Processing	Informatics	Semester	168	T: 28; PL: 28	6	Mandatory

2nd year/ 2nd semester

Course	Scientific area	Period	Hours	Hours at school	ECTS	Obs
Master Thesis (a)	Mathematics	Semester	840	OT: 42	30	Mandatory
Internship with Report (a)	Mathematics	Semester	840	OT: 14; E: 26	30	Mandatory
Project Work (a)	Mathematics	Semester	840	OT: 42	30	Mandatory

(a)Students may choose to do a Master Thesis, an Internship with a Report or a Project Work

Objectives and subject matter of each course

The exposition of the subject in theoretical-practical classes allows the student to understand concepts of actuarial mathematics applied to the life contingencies, as well as the practical use of the acquired concepts. The application of theoretical concepts in the resolution of exercises, allows students to develop capacities to conceptualize and solve complex problems, culminating in the field of study. The computational resolution of real cases (in class and outside of class) prepares students to solve practical problems in their future professional activity. The lessons being held in a computational lab will ensure contact with easy to find informatic applications (Excel and R Project).

Life Contingencies

1. Survival Models
    1.1. The future lifetime random variable
    1.2. The force of mortality
    1.3. Mortality Tables
    1.4. Extra Risks/Reduction of Risks
    1.5. Probabilities of death and survival for fractional ages
    1.6. Probabilities of death and survival for groups of individuals
2. Life Annuities
    2.1. Actuarial Present Value
    2.2. Annual Life Annuities with Constant Terms
    2.3. Increasing/Decreasing Life Annuities
    2.4. Annuities payable m times per year
    2.5. Annuities for groups of individuals
    2.6. Reversible Life Annuities
    2.7. Amortization scheme Annuity Certain
3. Life Insurance
    3.1. Introduction
    3.2. Life Insurance in case of life
    3.3. Life Insurance in case of death
    3.4. Mixed Life Insurance
4. Premium Calculation
    4.1. Types of Premiums
    4.2. The actuarial equivalence principle
    4.3. Leveled Premiums
    4.4. Reimbursement of Premiums paid
    4.5. Expenses

Initially, students are introduced to the calculation of probabilities concerning the death or the survival of one or more people. These two last points are combined to provide the base of understanding and calculation on life annuities, one of the types of life insurances products. Then, general concepts of a life insurance are introduced. At the end, the basic types of life insurances are presented in case of death, as well as the calculation of the actuarial values of these life insurance products: the premiums. We will calculate unique and leveled premiums as well as net and gross premiums.

Risk Theory I

1. Claims Distribution:
   1.1. Distributions for frequency (classes (a, b, 0) and (a, b, 1), composite models, mixed distributions, exposure effect)
   1.2. Distributions for severity
   1.3. Impact of changes in coverage on frequency and severity (franchising, capital limits, inflation)
   1.4. Estimation
2. Individual Risk Model
   2.1. Characterization and results
   2.2. De Pril's recursion
   2.3. Kornya Method
3. Collective risk model
   3.1. Hypotheses of the model
   3.2. The collective versus the individual model
   3.3. Main results
4. Selection of models
   4.1. Representation of data and model
   4.2. Graphical comparison of densities and distributions
   4.3. Testing the models
   4.4. applications
5. The distribution of aggregate claims
   5.1. The aggregate distribution
   5.2. Recursive method
   5.3. Numerical methods
   5.4. Approximations
   5.5. Impact of policy changes
   5.6. Risk measures for aggregate claims
   5.7. Applications

First, we review the distributions for the number and for the amount of claims in order to present the individual and collective risk models. These models are then characterized, either theoretically and statistically, and some methods of calculating probabilities associated with aggregate compensation are studied, ie the total amount of compensation that may occur in a policy portfolio during a given time.

Stochastic Processes and Modeling

1. Time Series:
   1.1. Order 2 processes, stationary processes, spectral representation
   1.2. Autoregressive (AR); Moving average (MA); Autoregressive moving average (ARMA); Autoregressive integrated moving average (ARIMA); Vector autoregressive (VAR)
   1.3. Estimation, periodograms, maximum likelihood
   1.4. Prediction
2. Markov chains in continuous time:
   2.1. Homogeneous Markov processes, Kolmogorov equations
   2.2. Transition Probabilities and Chapman-Kolmogorov Equation
   2.3. Stationary Distribution
   2.4. Non-Homogeneous Markov Processes, Matrix of Intensities, Kolmogorov Equations
   2.5. Limit Theorems
   2.6. Estimation
3. Diffusion Processes:
   3.1. Brownian or Wiener process: construction and properties
   3.2. Itô's Stochastic Integral: construction and properties; Itô's formula and applications
   3.3. Stochastic Differential Equations: existence and uniqueness of strong solutions
   3.4. Geometric Brownian Processes, Vasicek, Ornstein-Uhlenbeck, Cox-Ingersoll-Ross
   3.5. Diffusions; essential properties in dimension one.
   3.6. Estimation
4. Applications

Apply the concepts and properties of: stationary, integrated, univariate time series, stationary random series, filter applied to a stationary random series, backwards shift operator, backwards difference operator, roots of the characteristic equation of time series, multivariate autoregressive model.

Outline the processes of identification, estimation and diagnosis of a time series, the criteria for choosing between models and the diagnostic tests that might be applied to the residuals Develop deterministic forecasts from time series data.

Formulate the Chapman-Kolmogorov equations, calculate the stationary distribution and apply Markov chains as a tool for modeling and in simulation.

Apply the main concepts of Wiener Process. Show working understanding of stochastic differential equations, Ito integral, diffusions and mean reverting processes. Ito’s Lemma and proof, apply it to write down the stochastic differential equations for important processes and solve it.

Non-Life Insurance Pricing

1. A priori ratemaking

    1.1. Credibility Theory
        1.1.1. Bayesian credibility: Introduction and Poisson-Gamma Model
        1.1.2. Buhlmann and Bulhmann-Straub Models
        1.1.3. Jewel's Hierarchical Models
    1.2. GLM in a priori ratemaking
        1.2.1. The Exponential Family
        1.2.2. Model formulation
        1.2.3. Model Selection and Parameters estimation
        1.2.4. Modeling Claim Frequency
        1.2.5. Modeling Claim Severity
        1.2.6. Building the pricing structure
        1.2.7. Logistic Regression in Pricing Context
2. A posteriori ratemaking
    2.1. A priori vs a posteriori ratemaking
    2.2. Bonus Malus Systems definition
    2.3. Transition Probabilities and Limit Theorems
    2.4. Evaluation of Bonus Malus Systems
    2.5. Optimal Premium Scales
    2.6. Alternative Approaches

Learn the basic techniques for pricing in non-life insurance. Distinguish a priori ratemaking from a posteriori ratemaking. Distinguish premium calculation from tariff construction. Know how to build a tariff for non-life insurance. Know how to implement a bonus malus system and evaluate its effectiveness.

Machine Learning

1. Introduction to Machine Learning
   1.1. Machine Learning paradigms: Supervised Learning, Unsupervised Learning and Reinforcement Learning.
2. Data
   2.1. Types of Data
   2.2. Measures of similarity and dissimilarity
   2.3. Data normalization and visualization
   2.4. Dimensionality reduction by Principal Component Analysis
3. Supervised Learning
   3.1. Regression
   3.2. Decision Trees
   3.3. Artificial Neural Networks
   3.4. Support Vector Machines
   3.5. Graphical models
   3.6. K-nearest neighbour classifier
   3.7. Methods for classifier evaluation and comparison
   3.8. Ensembles
4. Unsupervised Learning
   4.1. Partitional clustering
   4.2. Probabilistic clustering
   4.3. Partitional Fuzzy clustering
   4.4. Hierarchical clustering
   4.5. Markov chain
   4.6. Clustering evaluation methods
   4.7. Other unsupervised learning topics

Understand the paradigms and challenges of Machine Learning, distinguishing Supervised, Unsupervised and Reinforcement learning. Learn the fundamental methods and their applications in data oriented. knowledge discovery. Understand data features, the selection of models and their complexity. Understand the advantages and disadvantages of the different methods. Interpret and evaluate experimental results. Validate and compare different Machine Learning algorithms.

Advanced Life Contingencies

1. Policy values
   1.1. Definition
   1.2. Methods
   1.3. Continuous time model
2. Policy alterations
3. Universal Life Model
4. Unit Linked Model
5. Estimation procedures for lifetime distributions
6. Describe and apply the Lee-Carter model for forecasting mortality
7. Graduation and graduation tests

Know the importance of technical provisions as well as the various calculation methods and be able to apply them to classic products in discrete and continuous time. Know the consequences of policy changes, eg in the calculation of new insurance sums and surrender values. Discussion of the Universal Life and Unit Link models. Learn to implement models of estimation of the future life time and application of the Lee-Carter model to estimate the mortality of a population. Use of graduation techniques in mortality models.

Risk Theory II

1. Premium principles
   1.1. Utility theory
   1.2. Premium calculation principles
   1.3. Properties
2. Reinsurance
   2.1. Quota share reinsurance
   2.2. Surplus reinsurance
   2.3. Excess of loss reinsurance
   2.4. Stop loss reinsurance
3. Ruin theory
   3.1. Continuous time model (the adjustment coefficient, ruin probability, Lundberg's inequality, maximum aggregate loss, approximations to the ruin probability)
   3.2. Discrete time model (the adjustment coefficient, ruin probability, Lundberg's inequality)
   3.3. The impact of reinsurance

To know the main principles of premium calculation and its calculation. To know the main reinsurance treaties and their influence on the aggregate claims. To calculate ruin probabilities, exact or approximated, in continuous or discrete time for some simple problems

Claims Reserving

1. Introduction to Technical Provisions.
2. Claims Reserving.
3. IBNR and IBNER Concepts.
4. Considerations about Inflation.
5. The run-off triangles and the ultimate.
6. Deterministic models (link ratio, grossing up).
7. Stochastic models (Thomas Mack, bootstrap, credibility models, GLM based models, others).
8. Presentation and interpretation of results for Solvency II.

At the end of this unit the student will have acquired the knowledge, skills and competences that allow her/him to evaluate the claim reserving of a line of business in non life insurance. Distinguish IBNR concept from IBNER. Understand the impact of inflation on data. Understand each method presented. Choose the best method to evaluate the claim reserving. Report the results in the Solvency II environment.

Actuarial Risk Management

1. The Risk Environment
   - 1.1. Apply the concepts of the actuarial control cycle to the risk management process.
   - 1.2. Explain the concept of enterprise risk management (ERM).
   - 1.3. Analyze aspects of the operating environment and their relevance to the ERM process.
   - 1.4. Explain the main differences between regulatory and economic capital.
   - 1.5. Define risk appetite and risk culture explain the importance of attitudes towards risk of key stakeholders.
   - 1.6. Evaluate the elements of an ERM framework for an organization.
2. Risk Identification
   - 2.1. Explain the purposes of risk classification.
   - 2.2. Explain the difference between uncertainty (immeasurable) and risk (measurable).
   - 2.3. Describe and classify different types of risk including financial risk, insurance risk, environmental risk, operational risk and business risk.
   - 2.4. Explain how the design of different products and services affects the risk exposure of the parties to a transaction and analyze the exposures for a particular transaction.
   - 2.5. Explain how the characteristics of the parties to a transaction affect the nature of the risk borne by each and analyze the exposures for a particular transaction.
   - 2.6. Explain the concept of risk pooling and the portfolio approach to the overall management of risks.
3. Risk Measurement and Modelling
   - 3.1. Explain the use of models for risk management in the context of:
     - 3.1.1. Pricing
     - 3.1.2. Reserving
     - 3.1.3. Valuation
     - 3.1.4. Capital management
   - 3.2. Describe different methods of risk aggregation and explain their relative advantages and disadvantages.
   - 3.3. Apply these models to practical problems in insurance, pensions or an emerging area of actuarial practice.
4. Risk Mitigation and Management
   - 4.1. Explain the most common risk mitigation and management techniques:
     - 4.1.1. Avoidance
     - 4.1.2. Acceptance
     - 4.1.3. Reduction
     - 4.1.4. Transfer
     - 4.1.5. Monitoring.
   - 4.2. Describe the principles of asset/liability management and apply them to the main types of liability held by financial institutions.
   - 4.3. Analyze the risk management aspects of a particular business issue and recommend an appropriate risk management strategy.
5. Explain the implication of risk for capital requirement, including economic and regulatory capital requirements.

Identify the risks that can affect an organization. Quantify the risks and their implications in the short and long term.Identify and quantify the cost / benefit ratio of risk mitigation techniques. Identify and document the expected results. Relate financial and non-financial variables in risk modeling and mitigation processes; such as the social and environmental impact of rising global temperatures. Produce documentation to integrate risk analysis into the decision-making process. Communicate risks to decision-makers in a technically documented and effective manner.

Students begin to learn the key risks that can affect an organization through an ERM framework. They learn to quantify risks and their short- and long-term implications as well as to identify and quantify the cost/benefit ratio of risk mitigation techniques. The ERM framework guides students to identify and document expected results. Models are presented for Relating financial and non-financial variables in risk modeling and mitigation processes; such as the social and environmental impact of rising global temperatures, with the ability to produce documentation to integrate risk analysis into the decision-making process and to communicate risks to decision-makers in a technically documented and effective manner.

Pension Funds and Social Security

1. Social Security
2. Social Security Three pillar system
3. Main systems and problems of the Social Security
4. Pension plans
5. Portuguese Legislation
6. Implementing a Pension plan
7. The importance of the hypotheses in the evaluation of liabilities
8. Evaluation of Liabilities
9. Actuarial methods of funding
10. Defined contribution pension plans
11. Some topics on management
12. Actuarial Gains and Losses
13. Risk measures Value at Risk and Tail Value at Risk in Pension Funds
14. ALM models

At the end of this course the student will have acquired knowledge, skills and powers to:
Identify subsystems of Portuguese Social Security (Social Security)
Calculate the pension for retirement and disability in accordance with the applicable legislation.
Major Systems and Social Security issues.
Define and explain the theory of the three pillars.
Set plans and pension funds.
Understand the concepts related with pension plans.
Elaborate a Pension Plan.
Understand the impact of the assumptions in the actuarial evaluation of benefits and future contributions.
Assess actuarial liabilities and contributions in a Pension Plan (Defined Benefit and Defined Contribution)
Distinguish the actuarial funding methods.
Understand the modus operands and management of pension funds.
Getting some knowledge of ALM.

Solvency Models

1. Specificities of the insurance sector versus the banking sector.
2. Solvency II and insurance risk management.
3. European Solvency II and how the solvency of an insurance undertaking is handled with the three pillars of Solvency II. Comparison of prudential supervision in Basel III.
4. The three pillars of Solvency II:
   4.1. Quantitative requirements (Pillar I): Solvency Capital Requirement (SCR) and Minimum Capital Requirement (MCR).
   4.2. Qualitative requirements and Supervisory process (Pillar II).
   4.3. Transparency and dissemination of information (Pillar III).
5. Capital requirements: models and evaluation:
   5.1. Market risk
   5.2. Credit risk
   5.3. Operational Risk
   5.4. Liquidity Risk
   5.5. Subscription Risk

The students start to learn the difference between uncertainty and risk and then proceed to the presentation on the Solvency II and Basel III frameworks. At this point, the specificities of each framework are presented with similitudes and differences discussed. With the main focus on Solvency III, the principle of the 3 Pillars is presented and explained. Some emphasis is given to the Pillar I Quantitative requirements with the two main risk measures associated to V@R are presented, explained and discussed, namely Solvency Capital Requirement (SCR) and Minimum Capital Requirement (MCR). The standard model is used to help in the discussion of the models used to evaluate the main risks; Market risk; Credit risk; Operational Risk; Liquidity Risk and Subscription Risk, marginally and jointly.

Health and Long Term Care Insurance

1. Introduction to Health Insurance
   1.1. Accident Insurance
   1.2. Sickness Insurance
   1.3. Disability Insurance
   1.4. Long Term Care Insurance
2. Multi-state Models
   2.1. Maximum Likelihood estimators for transitions between states
3. Actuarial Models for Sickness Insurance
   3.1. One-year cover and multi-year cover
   3.2. Risk factors and rating classes
   3.3. Premiums and Reserves
4. Actuarial Models for Disability Insurance
   4.1. Multistate Models for Disability Insurance
   4.2. Disability Annuities
   4.3. The Expected Time Spent in Disability
   4.4. Premiums and Reserves
5. Actuarial Models for Long Term Care
   5.1. Multistate Models in LTC
   5.2. Estimation of transition probabilities according to age
   5.3. Premiums and Reserves
   5.4. Mortality and Longevity Risks

Initially the basic concepts of insurance products covering the risk of sickness are introduced. Using concepts studied in previous UCs, the risk assessment of these products will be carried out, identifying suitable models to measure the risk involved. Actuarial premiums and reserves will be calculated. It will be privileged the resolution of practical cases in computational environment.

Investments Theory

1. Objectives of individual and institutional investors
2. Financial markets and types of financial investment: fixed and variable income assets
3. Models for determining the optimal investment portfolio:
   3.1. Factor model
   3.2. Financial asset balance model: CAPM
   3.3. Arbitration model: APT
4. Performance evaluation of financial asset portfolio management
5. Pricing (valuation) of financial assets: bonds, shares, options
   5.1. Financial markets free of arbitrage and complete markets; examples: binomial model and Black-Scholes model.
   5.2. Other stochastic models (e.g. diffusions) for pricing derivative financial products
6. Investment risk management. Risk measures: V@R, TV@R, EV@R

Describe the characteristics of forwards, futures, options and swaps and of the markets in such investments and of the main investment assets and of the markets in such assets.
Use the Capital Asset Pricing Model and or a multifactor model to calculate the required return on a particular asset, given appropriate inputs, and hence calculate the value of the asset. Explain the concepts of: efficient market, complete market, no-arbitrage, hedging.
Apply the risk-neutral or state price deflator approaches to valuing derivative securities and apply them. Use the properties of various stochastic models of the term structure of interest rates. Explain the limitations of the models described above and describe attempts to address them.
Use mean-variance portfolio theory to calculate an optimum portfolio and describe the limitations of this approach. Use mean-variance portfolio theory to calculate the expected return and risk of a portfolio of many risky assets, given appropriate inputs.

Systems for Big Data Processing

1. Overview
   1.1. Motivation, Applications
   1.2. Challenges
2. Programming models
   2.1. Batch vs. Incremental vs. Real-time
   2.2. Structured data vs. Unstructured data
   2.3. Declarative programming vs. General-purpose
3. Data storage
   3.1. Distributed file systems (e.g. HDFS)
   3.2. Relational databases
   3.3. NoSQL databases (e.g. key-value stores, document stores)
   3.4. Integration of multiple data sources (e.g. Hive)

4. Generic processing platforms
   4.1. Infrastructure: context, properties and implications
   4.2. Map-reduce model and supporting platform (e.g. Hadoop)
   4.3. Second generation platforms (e.g. Pig, Spark)
5. 5.Processing for specific domains
   5.1. Machine learning libraries (e.g. Spark MLlib)
   5.2. Platforms for graph processing (e.g. GraphX)
6. 6.Introduction to real-time processing platforms
   6.1. Data sources (e.g. Flume, Kafka)
   6.2. Data models: micro-batch vs. continuous
   6.3. Processing platforms (e.g. Storm, Spark Streaming)

This course will focus on the programming models and their use to solve concrete problems.

The main goals are the following: Know the different facets of processing large volumes of data. Know the main classes of systems for storage of large volumes of data. Know the dominant programming models for Big Data. Know solutions for specific problem domains. Be capable of identifying the best system class for solving a specific problem. Be capable of coding a specific problem solution in the most suitable programming model - Be capable of executing a big data application in a distributed platform.