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Original article

The sound of bronze: Virtual resurrection of a broken medieval bell

Vincent Debut[a,*], Miguel Carvalho[a,b], Elin Figueiredo[c], José Antunes[b], Rui Silva[c]

[a] Instituto de Etnomusicologia, Música e Dança, Faculdade de Ciências Sociais e Humanas, Universidade NOVA de Lisboa, Av. de Berna, 26 C, 1069-061 Lisbon, Portugal
[b] Centro de Ciências e Tecnologias Nucleares, Instituto Superior Técnico, Universidade de Lisboa, Estrada Nacional 10, Km 139.7, 2695-066 Bobadela LRS, Portugal
[c] CENIMAT/I3N, Departamento de Ciências dos Materiais, Faculdade de Ciências e Tecnologia, Universidade NOVA de Lisboa, 2829-516 Caparica, Portugal

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ABSTRACT

The bell from the church of S. Pedro de Coruche is one rare surviving example of early bells, cast during the 13th century in Europe, which was exhumed from a crypt-ossuary in an archaeological excavation carried out near Lisbon in Portugal. Of particular significance, it is believed to belong to a time period during which bell's profile has evolved noticeably, leading to bells with fine musical qualities and a well-defined sense of pitch. If the bell from Coruche was a tangible piece of evidence for tracing the history of bell casting in Europe, it had however lost all trace of its original sound: indeed the bell was found broken and incomplete and even if it has undergone a restoration process since the archaeological discovery, the use of an adhesive during the reassembly has changed somehow the vibrational properties of the bell structure. To bring back to life the sound of this broken musical artefact, a methodology combining experimental and numerical techniques from materials science and music acoustics is described in this paper. The general approach comprises material characterisation, geometrical measurements, modal analysis and physics-based sound synthesis techniques. By coupling a physical dynamical model of a bell impacted by a clapper with the modal properties of the original bell computed by Finite Element Analysis, realistic time-domain simulations of the Coruche bell dynamics were performed and realistic synthetic sounds were produced. As the original clapper has not survived, parametric computations have been performed to illustrate the changes in bell sounds associated with clappers of different mechanical properties. The overall approach provides insight into the tuning of this medieval bell which can be compared to the modern-type tuning, and reproduce the sound that the bell from Coruche might have had. The strategy developed can be easily adapted to other musical instruments in poor/variable states of preservation, therefore benefiting the importance of such non-renewable cultural resources.

1. Research aims

As a rare example of early bells cast prior to the introduction of bell tuning techniques, the bell from the church of S. Pedro de Coruche is a valuable historical heritage object which deserves special scientific attention from archeologists and conservators. Discovered fragmented and incomplete, the bell has been partially reassembled, thus restoring its morphological aspect, but the use of an adhesive during the restoration process combined with the loss of a fragment of the original bell have eliminated any trace of its original sound. However, restoring its sound qualities is possible by exploiting the theoretical models and experimental methods used to investigate the acoustics of musical instruments. The main aim of the present work is then to bring back to life the sound of the bell from Coruche, by developing an instrumental strategy to assess the acoustical properties of bells that are no longer in condition to be played. The "virtual resurrection" is achieved through physical modelling sound synthesis techniques, guided by acoustical considerations based on measurements and simulations, as well as using the actual bronze material properties identified from elemental analysis and microstructural examinations.

2. Introduction

Musical instruments are fascinating objects which are an important part of every culture. They relate history, culture, art and science, and through the music they are intended to play, they embody spiritual values of cultural heritage. According to time and

* Corresponding author.
E-mail addresses: vincentdebut@ctn.ist.utl.pt (V. Debut), miguel.carvalho@ctn.ist.utl.pt (M. Carvalho), emf12055@campus.fct.unl.pt (E. Figueiredo), jantunes@ctn.ist.utl.pt (J. Antunes), rjcs@fct.unl.pt (R. Silva).

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Fig. 1. The bell from the church of S. Pedro de Coruche, Portugal.

space, they have taken many forms and through the creative efforts of gifted crafters, primitive instruments became competent musical objects fulfilling cultural criteria for music performance.

Although bronze bells with remarkable musical qualities have been cast in China some 3000 years ago, it is only during the 17th century that the art of bell funding had reached its peak in Europe when the Hemony brothers successfully designed and cast the first precisely-tuned carrillon [1]. Improving on the basic search to relate simply the bell profile and the sounding frequencies, the Hemony brothers, with the collaboration of Jacob Van Eyck, discovered how to correctly shape the profile of bells so that the ratios of their vibrational frequencies fall into musical relationships. Besides giving strong tonal characteristics to their bells, this tuning notably enhanced their musical pitch so that both melodies and chords could be played from an ensemble of bells. Nevertheless, the so-called traditional *minor-third* bell has not emerged overnight and it is thought that the first experiments in bell casting in Europe go back to the 4th century. At that time, bells were certainly not intended to play polyphonic music. Instead, they were used to regulate the daily activities or to celebrate rituals for which the tuning qualities and musicality of bells were not of prime importance. Starting from the 12th century, a transition towards the design of new bells has occurred and the evolution of the bell's profile from conical to the actual curved shape has gradually provided bells with pleasant sounding characteristics [2].

Dated 1287, the bell from the church of S. Pedro de Coruche (Fig. 1) is therefore a rare surviving example of early bells belonging to that transition period. Of particular significance, it is the oldest recognised bell in Portugal and as such, its archaeological discovery in 2001 has raised many questions regarding the art of bell casting in the Iberian Peninsula. A first study focusing on historical and social aspects has been carried out by Sebastian [3] but while it also discusses aesthetical features of the bell, no information has been reported regarding its sound. The main reason for this comes from the poor state of preservation of the bell which was discovered broken and incomplete. Even if it has been reassembled since, a fragment of about 75 mm^2 is still missing near the head and the use of an adhesive during the reassembly process has modified somewhat the stiffness and damping properties of the overall structure, therefore preventing any reliable trace of its original sound.

To investigate the Coruche bell's acoustics, it is now possible to exploit the advances in computational modelling and simulations of musical instruments which have nowadays, in many respects, reached maturity. Crucial information about the important vibrational modes of the instrument components can be obtained from powerful computations and experimental techniques, while reliable and fast sound synthesis methods, based on the dynamical differential equations of the system, can be implemented for generating synthetic sounds. To revive the sound of the bell from Coruche, two common techniques of music acoustics are considered in this work:

- the Finite Elements Method (FEM) for assessing the vibrational properties of the bell structure;
- a modal synthesis technique which uses the FEM-computed modal data as inputs of the computational model for reproducing the bell's sound.

Even if this methodology appears straightforward and physically correct, the sound computed however crucially depends on the mechanical properties of the bell material, i.e. the Young's modulus, Poisson's ratio and density, which control the wave propagation within the structure. Knowing that bronze composition has changed over time, an elemental and microstructural analysis of the bell bronze material has also been performed in order to calculate and identify its mechanical properties, and thus bring back to life the original voice of this medieval bell as close as possible.

In this paper, the complete methodology developed to address the acoustical features of the bell from Coruche is described. As far as we know, such a multidisciplinary approach has never been attempted, at least for historical bells, and the method can be easily adapted to study other bells in a poor state of conservation, therefore contributing to the preservation of valuable cultural heritage. Of particular interest, the synthesis method, which is based on physical modelling techniques, allows many computations to be performed by simple changes of the model parameters. This study particularly benefits from this convenient feature of the computational approach, as it offers a way to counteract the unknown mechanical properties of the original clapper by performing parametric computations, therefore providing an overview of plausible sounds for this broken Medieval bell.

3. Material study of the Coruche bell

3.1. Background on bell material

Although there are examples of bells cast in different metals such as copper or iron, bells have been mostly produced in bronze (Cu-Sn alloy), to which superior sonorous qualities are generally attributed [4]. The bronze alloy used to cast a bell definitively influenced its acoustic features, its external appearance and durability. Compared to iron, bronze has a longer durability, especially when exposed to external atmospheric conditions. Compared to pure copper, the addition of tin allows an increase in hardness, preventing a deformation of the bell by the clamp which in turn would influence the tone. However, a compromise on the tin amount in the alloy had to be made, otherwise too much tin would turn the alloy too brittle, and thus easily breakable during striking. With respect to the sound, it has been found that by rising the tin content an increase in sound duration would be obtained, however, other elements that were commonly present in the alloy, such as lead, that was known to improve fluidity and thus casting properties, would produce a decrease in the sound duration [5]. Taking into account an optimum balance of the Cu-Sn alloy properties used to produce bells, a general bell bronze composition was early set up in the range of 20–25 wt.%Sn. Commonly described as four parts of copper to one of tin or three parts of copper to one of tin, the bell metal recipe can even be found in Theophilus's twelve-century treatise on bell casting. Nevertheless, analysis of bells from the Middle Ages until recent times have shown that Medieval bells did frequently have lower amounts of Sn (in the order of 10–15 wt.%Sn) and that Pb was frequently present in contents from 1 to 3 wt.% [4]. Due to

Table 1
Elemental composition of the bronze bell. Average of three spot analyses made by micro-EDXRF.

Cu	Sn	Pb	As	Fe	Ni
75.6 ± 3.6	21.3 ± 2.1	3.0 ± 1.6	< 0.1	< 0.05	nd

Results in wt.% and normalised; nd: not detected.

the relatively high economic value of bronze, frequent remelting of ancient or broken bells were performed in the past and even nowadays if significant historical and cultural value is not assumed, due to the constraint of wartime or even as a result of ignorance and vandalism. This has led to the loss of many bells, being scarse the number of Medieval bells still preserved.

3.2. Elemental and microstructural characterisation of the bronze

A small sample of the bell representative of a cross-section was taken at the top fracture with an electrical rotating saw. The sample was cold-mounted in resin and metallographically prepared, by grinding and polishing until one-quarter micron diamond size.

The sample was subjected to elemental analysis made by energy dispersive micro X-ray fluorescence (micro-EDXRF) in an ArtTAX Pro spectrometer [6]. Three spot analyses were made to account for material heterogeneities at the microstructural level, being considered the average values. Details on the experimental conditions and quantitative procedure have previously been published in detail [7]. The results of the elemental analysis showed that the bell was made of a bronze with around 21%Sn and 3%Pb (Table 1).

Microstructural observations were made by Optical Microscopy (OM) and Scanning electron microscopy with energy dispersive X-ray spectrometry (SEM-EDS). An inverted Leica DMI5000 M microscope, coupled to a computer with the LAS V2.6 software was used, and microstructure examinations were made in bright field, dark field and polarized light with the surface in as-polished state. Further details on this equipment and its application in the study of cultural metals can be found in [8].

SEM analyses were performed in a Zeiss model DSM 962 equipment which has backscattered electrons and secondary electrons imaging modes and an energy dispersive spectrometer from Oxford Instruments model INCAx-sight with an ultra-thin window able to detect low atomic number elements as oxygen and carbon. No conductive coating was performed, being used a conductive carbon tape bridging the sample to the ground, following an experimental procedure for cultural materials published previously in [9].

The microstructural study allowed the identification of a typical as-cast microstructure, with the presence of alpha (copper rich) phase dendrites and alpha + delta eutectoid, the presence of small Pb globules and small size pores dispersed in the microstructure as well as macropores mainly in the most internal regions of the cast. Also, interdendritic corrosion was identified developing from the external, internal and fracture surfaces towards the bulk (Fig. 2).

Images taken by OM and SEM-EDS at different magnifications were analysed with the freely available program ImageJ [10] for area/volume calculations of the different phases (Fig. 2). Both macro- and micropores were taken into account for the pore

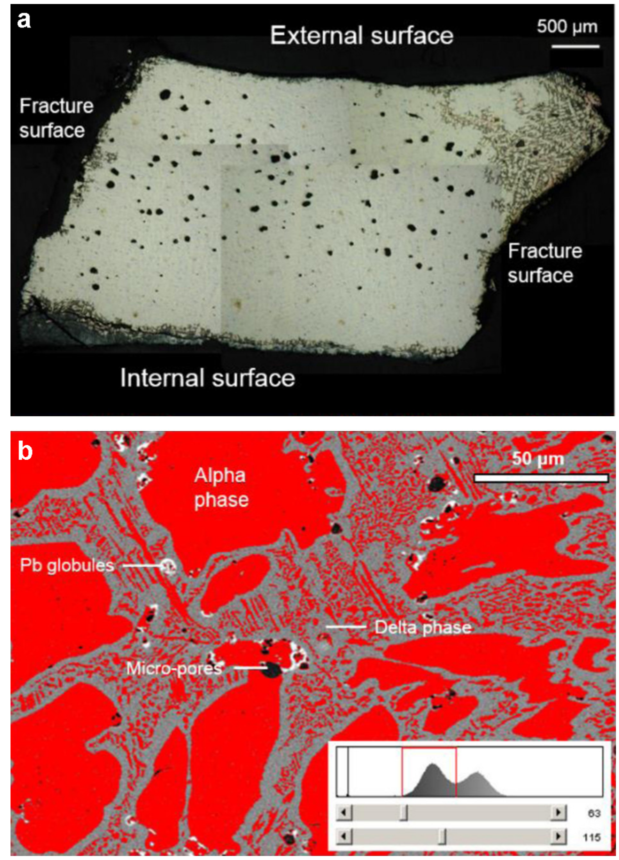

Fig. 2. Microscopic images of the microstructure of the bronze bell. (a) Bright field OM view of the studied cross-section (assembly of various pictures). Macropores can be easily distinguished at the most internal regions of the cast (spherical black shapes). Also, interdendritic corrosion at the areas near the external, internal and fracture surfaces are visible (dark grey colour, with a stronger incidence at the top right side). (b) Treated (SEM) BSE image using ImageJ software to calculate the amount of alpha phase (in red, and depicted in the original greyscale histogram at lower right) present in the alloy. Note also delta phase in light grey, micropores in black and lead globules in white.

analysis. At least five analyses were made for each phase being considered the average areas. Results are reported in Table 2.

3.3. Calculation of the density of the bell bronze

The density ρ of the bronze depends on the density of each phase ρ_i identified in the microstructure of the bronze bell (alpha, delta, Pb and pores) and its relative volume (V_i/V), as follows:

$$\rho = \sum_i \rho_i V_i / V; \quad V = \sum_i V_i \qquad (1)$$

The density of pores, Pb globules and delta phase can be assumed as 0, 11.34 and 8.68 g/cm^3 respectively, based on literature and X-ray diffraction database files (Table 2).

Accordingly to the metastable Cu-Sn phase diagrams [11], alpha phase can exist in a range of compositions from 0%Sn (pure copper) to 15%Sn (in an annealed microstructure), which has implications

Table 2
Relative volumes of each phase identified in the bell bronze and corresponding densities. The relative volume considering only solid phases (excluding pores) is also presented.

		V_i/V (%)	V_i/V solid phases (%)	ρ (g/cm^3)	Notes on ρ
Alpha phase	~8%Sn (sand casting)	52	53.33	9.03	Calculated from Eq. (2)
Delta phase	$Cu_{32}Sn_8$ (32.6%Sn)	45	46.15	8.68	From X-ray diffraction file PDF 71-0122
Pb globules	(Considering 100%Pb)	0.5	0.51	11.34	[12]
Pores		2.5		0	

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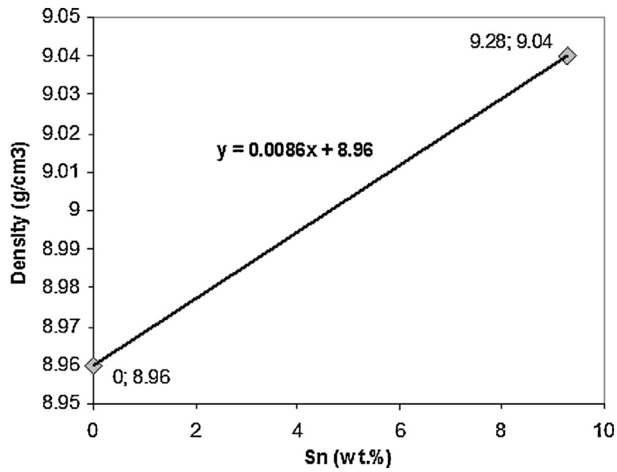

Fig. 3. Variation of the density of alpha phase according to its composition, in the range 0–9.28%Sn (8.960–9.034 g/cm^3), based in the X-ray diffraction files PDF 85-1326 (alpha-copper) and PDF 44-1477 (Cu-9.28%Sn).

in its density. The alpha phase of the bell bronze can be estimated to have a content of 8%Sn, the typical content for sand castings, as those used for casting bells. Since the density of such specific alpha phase was not available in literature or specific databases to the present knowledge of the authors, a linear equation (2) that takes into account the density of pure copper (alpha-copper) and alpha phase with 9.28%Sn (data taken from X-ray diffraction database files) (Fig. 3), was considered to obtain the density of alpha phase with a specific Sn content (X_{Sn}):

$$\rho_{alpha\ \ phase} = 0.0086X_{\mathrm{Sn}} + 8.96 \tag{2}$$

Using Eq. (2), an alpha phase with 8%Sn in the bronze has a calculated density of 9.03 g/cm^3.

Taking into consideration the densities and relative volumes for each phase exposed in Table 2 and using Eq. (1), the density of the bell bronze is found to be 8.66 g/cm^3.

3.4. Calculation of the Young's modulus of the bell bronze

3.4.1. Young's modulus of the solid phases

The bell bronze is composed by three solid phases (alpha, delta and Pb), excluding the pores. Taking into consideration the solid phases only, the Young modulus E_0 can be considered as depending on the Young modulus of each phase E_i and its relative volume (V_i/V), as follows:

$$E_0 = \sum_i E_i V_i / V \tag{3}$$

The Young modulus of some phases can be taken straightforward from literature, as Pb that can be considered as 15 GPa [12]. Also, the Young's modulus for alpha-Cu, and some Cu-Sn intermetallic phases as epsilon and eta can be obtained in specific literature [13–15] (Table 3). However, the value for the intermetallic delta phase has not been intentionally measured to the present authors' knowledge. Nevertheless, it can be inferred from the work of Subrahmanyam [16], as the lowest value measured, 80 GPa, that the author incorrectly attributes to the epsilon phase – lately characterised with much higher values (around 108 GPa) to alpha plus eta phases alloys by diverse authors [13,14]. Additionally, the Young modulus of alpha phase with 8%Sn is not available in literature, but it can be estimated through a linear equation

Table 3
Young modulus of different phases that can be present in a bronze alloy, based on literature and present calculations.

Solid phases of Cu-Sn		Young Modulus (GPa)	Notes
Alpha-Cu	100%Cu	124	[13]
Alpha ~8%Sn	Cu-8%Sn	113.2	From Eq. (4), see text
Delta	$Cu_{31}Sn_8$ Cu-32.6%Sn	80	From [16], see text
Epsilon	Cu_3Sn Cu-38.2%Sn	108.3	[13]
Eta	Cu_6Sn_5 Cu-61%Sn	85.5	[13]

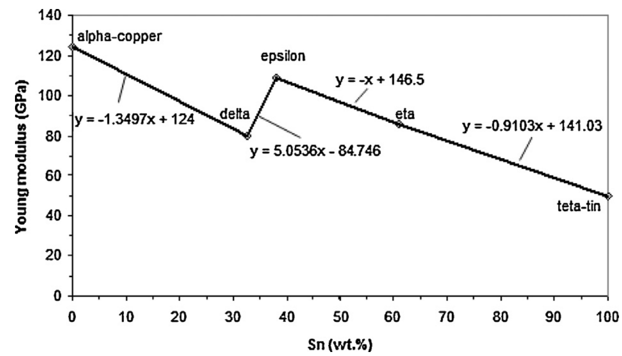

Fig. 4. Young modulus for pure copper, pure tin, and each intermetallic phase in Cu-Sn bronze alloy and equations for linear relationships within, calculated in the present work.

that considers the Young's modulus of alpha-copper [13] and delta phase (Fig. 4), as follows:

$$E_{alpha\ phase} = -1.3497X_{\mathrm{Sn}} + 124 \tag{4}$$

Based on Eq. (4), the alpha phase with 8%Sn has a calculated Young's modulus of 113.2 GPa. Finally, using Eq. (3), the Young modulus of the various solid phases can be considered as 97.37 GPa.

3.4.2. Young's modulus of the bell bronze considering the pores

Since the bell bronze is not totally composed by solid phases, the pores have also to be considered for the calculation of the Young's modulus of the bronze bell. Taking into consideration the work of Pabst and Gregorová [17], the following two equations can be used, assuming that the bell bronze can be considered as a homogeneous/dense material with spherical pores:

$$E_r = E/E_0 \tag{5}$$

where E_r is the relative Young's modulus, E is the effective Young's modulus of the porous material – this is, the unknown Young's modulus that one is trying to obtain – and E_0 is the Young's modulus of the matrix (dense, pore-free material), and

$$E_r = (1 - \phi) \times (1 - \phi/\phi_c) \tag{6}$$

where ϕ is the volume fraction of porosity and ϕ_c is the critical porosity volume or percolation threshold for spherical pores, which value can be considered as 0.289573 [18]. Taking into consideration Eqs. (5) and (6), the Young's modulus calculated for the bell bronze is 86.74 GPa.

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4. Physical-based modelling of the clapper-excited bell

4.1. General framework

As for every percussion instruments, the sound produced by bells results from structural vibrations. Since bell vibrations remain essentially linear [19], the spectrum of bell sound is entirely determined by its vibrational modes which all combine together to create a resultant pitch.[1] However, nonlinear effects might arise during the short excitation, resulting in physical phenomena of significant musical importance such as the timbral changes observed for different amplitudes of excitation. Other prominent feature in the sound of bells is the warbling effect which manifests as a low-frequency beating pulsation modulating the sound amplitude, which is a consequence of the frequency split from degenerate modal pairs due to the presence of slight asymmetries in the bell shape.

For sound synthesis purpose, the modelling approach devised in this work is based on a modal representation of the system dynamics. It consists in expressing the bell response to a given excitation as the sum of the responses of each bell mode. For realistic simulations, this technique requires a good knowledge of the system modal parameters, i.e., the modal frequencies, modal damping, modal masses and modeshapes, which characterise the modes of vibration. In practice, modal parameters can be asserted either experimentally or numerically, respectively through modal testing or eigenvalue computations. In this work, since the complete original bell has not survived, the numerical approach has been favoured and a Finite Element analysis, which involves a discretization of the bell structure in terms of a large number of interconnected small elements, has been performed. Further details on bell vibration and sound synthesis of musical instruments appear in [1,20–23].

4.2. Bell geometry description

The studied bell is a small medieval bell of 21 cm in height to its shoulder, with weight of 4.8 Kg (without the crown), and with a diameter of about 21.7 cm at the rim. Since bells are structures of variable thickness and radius along their height, a method enabling the reconstruction of the bell profile from a series of geometrical measurements at some grid points, including mean radii, absolute position and thickness measurements, has been developed. A mesh of 936 points was defined, with 36 points regularly spaced azimuthally at 26 height locations. The bell stood on a rotating support, with its head down, and profile measurements were performed for every 10-degree over the bell circumference. Absolute position of measurement points in the outer surface were contactless, using a single point optical displacement transducer (Bullier Automation, ML5 model), and radii and horizontal thicknesses were measured using a slide gauge as illustrated in Fig. 5. Typical errors in measurements are less than ± 0.15 mm.

The locations of the grid points in three-dimensional space were then computed from the geometrical data. Since absolute position measurements only describe deviations from a reference surface, a centerline had to be defined in order to assemble the 26 successive cross-sections along the height of the bell. The mean radius and corresponding deviations of each bell "ring" were estimated from the measured diameters and absolute position data, and then deviations were minimized from the mean radius using a least-squares procedure. Care was also taken to accommodate possible errors due to offset alignment of the bell on the support. Once the bell radius was known for each cross-section, the complete bell is simply reconstructed as a pile of rings, by assuming that all cross-sections are based on a common center. Finally, simple geometric transformations allow the description of both inner and outer profiles of the original bell.

Fig. 5c shows the actual inner and outer radii over the bell circumference. Not surprisingly, the bell geometry shows deviations from a perfect axisymmetric structure, with maximum deviations of about 2.5 mm. As previously mentioned, such deviations originate the frequency splitting from the degeneracy of the bell structural modes, one side-effect being the presence of beats in the bell sound (Fig. 7). It is worth noting that radii estimations can be partially perverted by some corrosion phenomena and partial loss of corrosion layers. Typically, corrosion for archaeological bronze can reach depths of 500 μm [24,25] (see also in the sample taken in this work in Fig. 2, where the thickest corrosion layer observed, at bottom-left, displays approximately of such thickness). A possible detachment of such a corrosion layer would lead to measurements 0.5 mm below the original surface. On the other hand, if corrosion products related with chlorides are present ("bronze disease"), then it is expected the formation of "blisters" [25] being the measurements on those areas far above the original surface. However, the influence of corrosion in the present results is certainly limited, as care has been paid to measure grid points with no apparent significant surface alteration. In addition, note that the deviations are about one order of magnitude higher than normal corrosion layer thicknesses.

4.3. Finite Element Modal analysis

From the knowledge of the bell geometry, a 3D model of the historical bell was constructed by means of Finite Elements. The Finite Element package Cast3M [26] developed by the CEA (France) was used to construct the FEM model as well as to carry out the numerical modal computation.

The mesh displayed in Fig. 6a consists of 28,512 solid elements (20-node hexahedral), which appears fine enough for accurate modal computations on the basis of preliminary convergence tests. Eigenvalue computations were performed as if the bell was loosely suspended, by assuming free boundary conditions. Based on the results of section 3, the Young's modulus, density and assumed Poisson's ratio were $E = 86.74$ GPa, $\rho = 8.66$ g/cm^3 and $\nu = 0.34$ respectively.

Because modelling the damping properties of structures remains challenging and also because damping is very small for bells, modal computations were performed using a conservative model. It means that undamped eigensolutions were used to define the modal basis of our physical model. However, to assert the bell dynamical responses accurately, energy dissipation will be accounted in our modelling by assuming that damping is proportional in the system, so that mode shapes are real, a legitimate approximation for low damped system. Vibrational and acoustical dissipations were then expressed in terms of modal damping values by using tentative estimates obtained from a detailed modal identification of a modern laboratory bell. Notice that modal damping in bells stems in part from material damping, but mostly from sound radiation which depends on the modal frequencies and modeshapes. Therefore, using a modern bell for asserting dissipation is a credible option. Previous identification results showed that modal damping increases with frequency, with values ranging from 0.0002 (for the lowest mode) to 0.001. From the computational point of view, incorporating frequency-dependent damping effects can be easily achieved using the modal approach devised in this work.

[1] What the perceived pitch of a bell is still constitutes a questionable issue which is not addressed here. Instead, attention is on the frequency relationships between the vibrational modes of the bell which are indicative of the bell "internal" tuning as well as of the understanding of the art of bell casting by founders.

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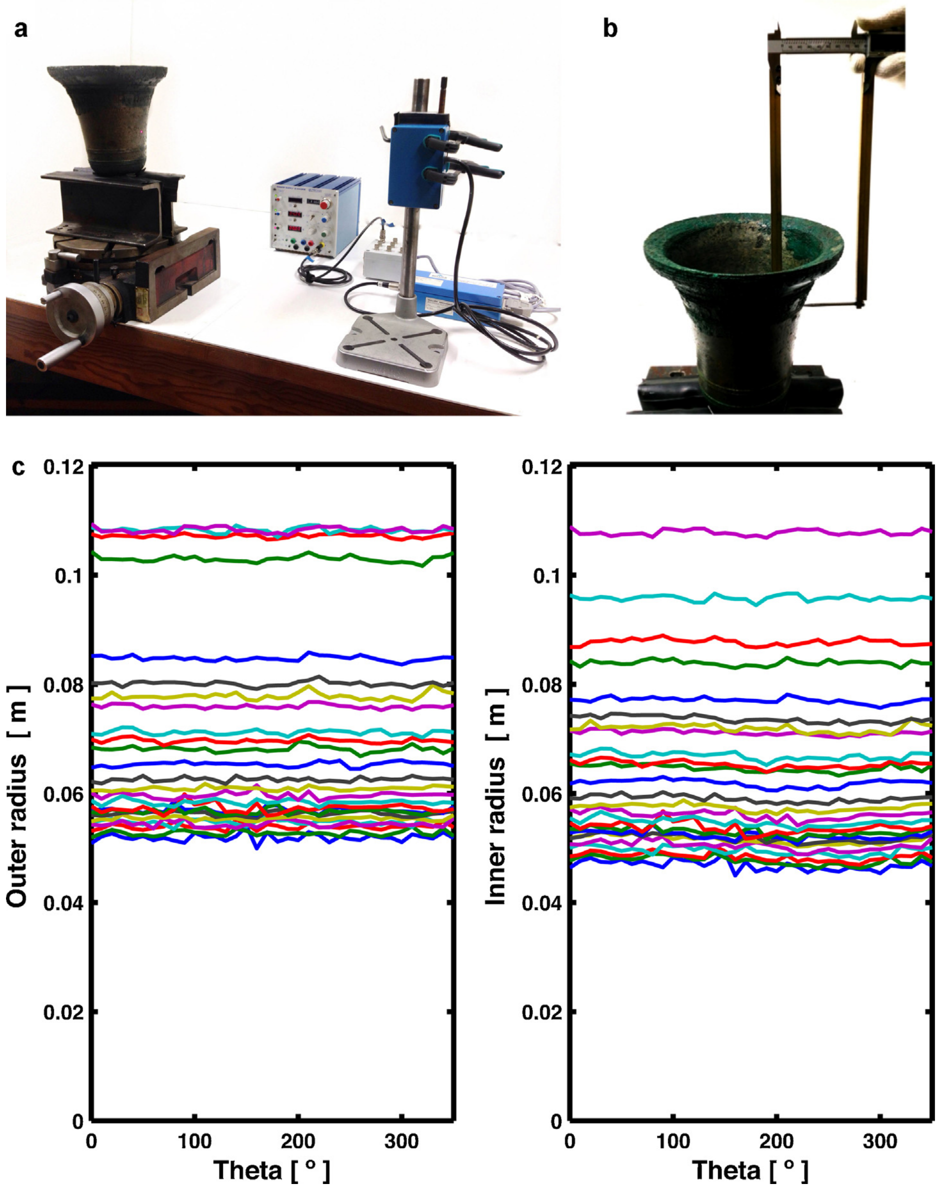

Fig. 5. Bell geometry description. (a) Outer surface measurements (b) horizontal thickness measurements and (c) measured outer (left) and inner (right) radii along the bell circumference.

Fig. 6b presents the mode shapes of the first five partials stemming from FEM computations (although computed, rigid body modes which correspond to translation and rotation motions of the undeformed structure are not relevant here). As expected for typical axially symmetric structure, normal modes appear in degenerate pairs so that one bell partial is constituted by a pair of modes. However, because slight asymmetries are present in the bell shape (Fig. 5c), one notices small differences in frequency between the two modal components of each partial. Perceptively, the frequency heard of a given partial corresponds to the mean of two modal frequencies, with a variation of its amplitude at a rate given by the frequency difference. Of particular interest, Table 4 presents the relative frequency ratios of each partial of the Coruche bell compared with the typical values of a modern minor-third tuned bell by assuming the first partial as the reference. It can be observed that except for the second and fourth modes, differences smaller than 3% are evidenced among the other three partials. In spite of suffering from poor tuning compared to what European founders will later believe to be the ideal tone, this historical bell has three partials reasonably in tune, which would give the Coruche bell a rather strong sense of pitch when struck.

Table 4
Relative frequency ratios of a tuned traditional minor-third bell and of the bell from Coruche.

	Relative frequency ratio to hum				
	f_1/f_1	f_2/f_1	f_3/f_1	f_4/f_1	f_5/f_1
Minor-third tuning	1	2	2.4	3	4
Coruche bell tuning	1	1.7	2.3	2.6	3.9

A quantification of the amount of beats in the bell sound can also be done by examining the frequency differences between modal-pair components. By doing so, we quantify the so-called *warble* of a bell which is perceived as a low-frequency pulsating beat in the sound radiated and which strongly affects our perception of bell sounds [1,27]. Obviously, the significance of beats in the sound is directly related to other characteristics of the bell, in particular the damping and radiation efficiency of each partial. As displayed in

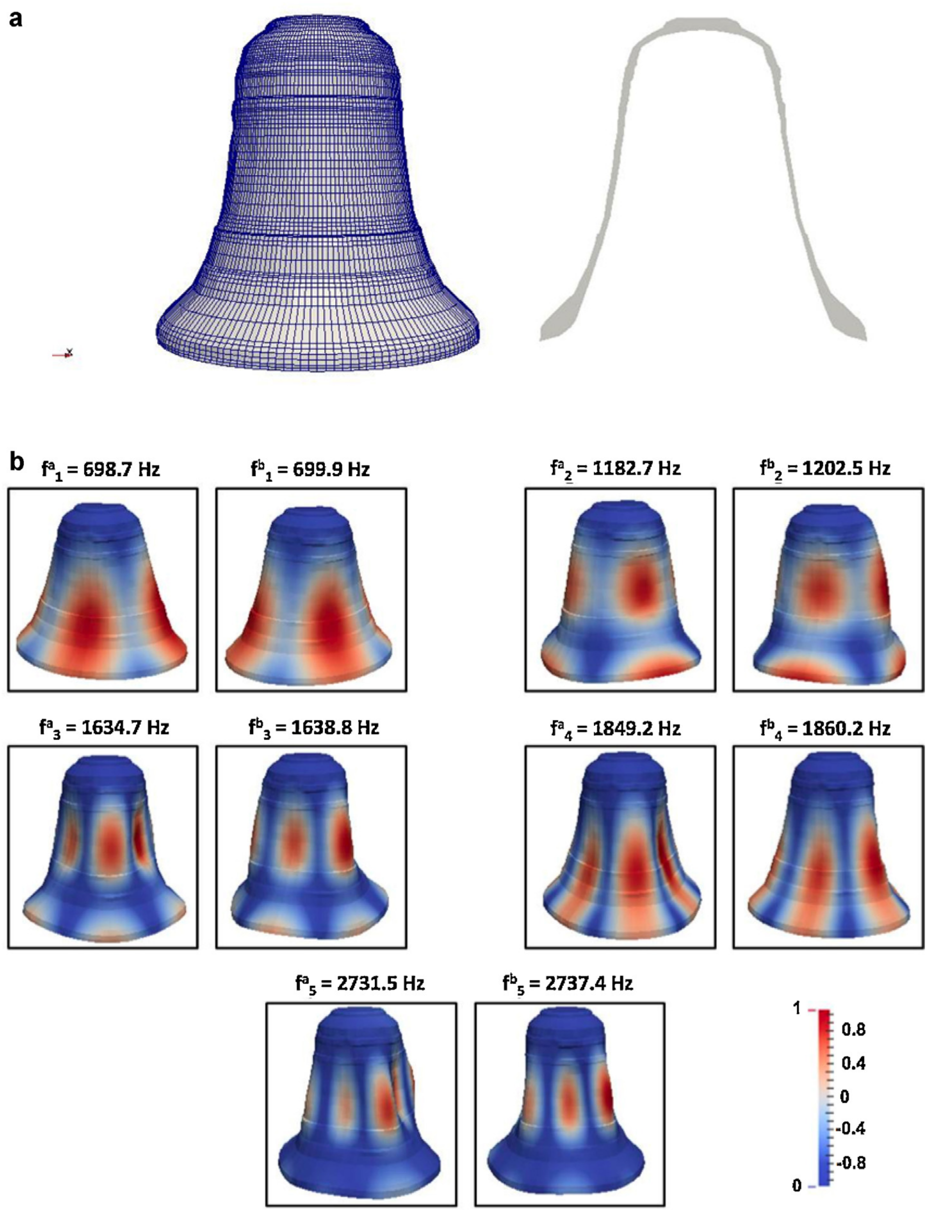

Fig. 6. Finite Element Modal Analysis. (a) Mesh and vertical cross-section in a single plane. (b) Computed modeshapes for the five lowest partials.

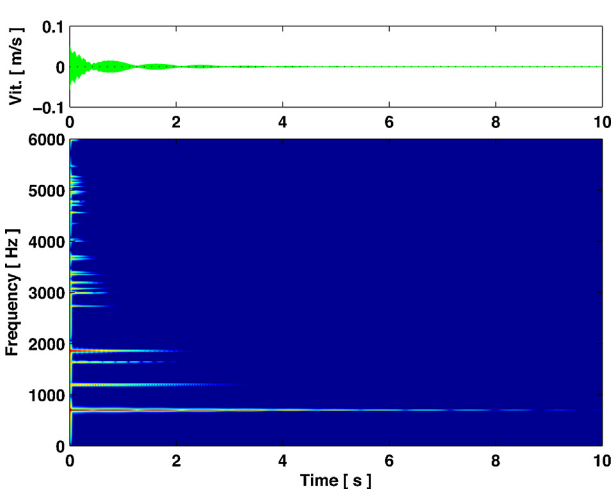

Fig. 7. Computed vibratory response of the Coruche bell velocity at the impact location. m_c = 0.2 Kg, r_c = 0.01 m, ν_0 = 0.1 m/s and h_c = 0.0475 m.

Fig. 6b, FEM computations showed that the warble ranges from 1 to 20 Hz, while a value of less than 1 Hz is typically required for today good quality bells [27]. As the warbling effect is a consequence of the difficulties of casting perfect axisymmetrical bells, this rather wide distribution of beating frequencies illustrates the technical limitations of the bell founder at the time.

4.4. Formulation of the clapper-excited bell dynamics

We now turn to the description of the nonlinear physical modelling of a generic bell impacted by a clapper and present the dynamical equations. A modal synthesis technique is used to describe the system dynamics and to numerically obtain the actual solutions, a modal discretization of the dynamical equations is performed in terms of the bell modal properties. Briefly, the physical model includes:

- a set of ordinary differential equations which represents the multi-modal vibrational behaviour of the bell structure;

- a dynamical equation for the clapper motion;
- an excitation force which stems from a contact model for the bell/clapper nonlinear interaction.

4.4.1. Bell dynamics

In a modal framework, the physical motion of the bell at generic position $\mathbf{r}=(r, \theta, \mathbf{z})$ is assumed to be given by

$$y(\mathbf{r}, t) = \sum_{n=1}^{N} q_n(t)\varphi_n(\mathbf{r}) \quad (7)$$

where $\varphi_n(\mathbf{r})$ represents the mode shape of the n-th mode of the bell and $q_n(t)$ represents its corresponding modal amplitude.

The wave equation of the bell motion is then replaced by a set of N second-order equations which govern the time-dependent amplitudes of each mode, according to:

$$m_n\ddot{q}_n(t) + c_n\dot{q}_n(t) + k_n q_n(t) = F_n(t) \quad (8)$$

where m_n, $c_n = 2m_n\zeta_n\omega_n$ and $k_n = m_n\omega_n^2$ are the system modal masses, damping values and stiffnesses respectively, ω_n being the circular modal frequency and ζ_n the modal damping value which accounts for both internal material and acoustical dissipation.

The modal forces $F_n(t)$ are obtained by projecting the external force $f(\mathbf{r}, t)$ on the mode shape $\varphi_n(\mathbf{r})$ of the modal basis, as:

$$F_n(t) = \int_r \mathbf{f}(\mathbf{r}, t)\varphi_n(\mathbf{r})\,d\mathrm{r} \quad (9)$$

so that, for the case of a localized radial impact excitation at $\mathbf{r} = \mathbf{r_0}$, Eq. (9) becomes

$$F_n(t) = \int_r -f_c(t)\delta(\mathbf{r}-\mathbf{r}_0)\varphi_n(\mathbf{r})d\mathrm{r} = -f_c(t)\varphi_n^R(\mathbf{r}_0) \quad (10)$$

where $f_c(t)$ is the contact force, $\varphi_n^R(\mathbf{r}_0)$ is the radial component of the mode shape $\varphi_n(_{\mathrm{r}0})$, and minus sign comes from the convention that positive force acts toward positive radial direction.

4.4.2. Clapper dynamics

The model for the excitation has now to be completed by writing the clapper dynamics. The clapper is modeled as a rigid body of mass m_c, neglecting any elastic dynamics of the clapper beyond the contact deformation during the impact. Its normal (radial) motion is simply governed by the rigid-body dynamical equation:

$$m_c\ddot{z}(t) = f^R(\mathbf{r}_0, t) \quad (11)$$

where $f^R(\mathbf{r_0}, t)$ is the radial component of the external force.

4.4.3. Bell/clapper interaction force

The bell/clapper interaction force $f_c(t)$ must be determined during the contact time. To that end, a theoretical contact model between the bell and the clapper is needed. An elegant choice is given by the theory of linear elasticity developed by Hertz, referred to as the Hertz's law, for which the contact force, which acts perpendicular to the tangent plane including the contact point, is a nonlinear power function of the local compression [28]. The Hertz law expresses the interaction force $f_c(t)$ between two elastic contacting bodies as

$$f_c(t) = \mathrm{K_c}\delta(t)^b \quad (12)$$

where $\delta(t)$ is the relative normal deformation (compression) between the contacting solids and b is typically equal to 3/2 for a sphere in contact with an elastic half-space. K_c is a contact stiffness coefficient which depends on both surfaces geometries and the elastic properties of the two solids, given analytically by [29]:

$$\frac{1}{K_c} = \frac{3}{4}\left[\frac{1-\nu_1^2}{E_1} + \frac{1-\nu_2^2}{E_2}\right]\sqrt{\frac{1}{R_1} + \frac{1}{R_2}} \quad (13)$$

where E_1 and E_2 are the Young modulus, ν_1 and ν_2 the Poisson coefficients and R_1 and R_2 the radius of the two contacting bodies.

The simplicity of Eq. (12) makes Hertz's model particularly attractive for the present case. First, the model can be easily handled within the modal representation and second, its nonlinearity enables to properly reproduce the significant spectral changes observed in bell sounds when a bell is struck with different impact forces. In particular, for $b > 1$, it is obvious that the impact force increases with compression. For impacts of increasing energy, high-frequency modes will be progressively increasingly excited and consequently bell sounds will become brighter. Also, the contact stiffness K_c influences the contact time which controls the frequency spectra of both the excitation and the corresponding response. The harder the contact stiffness, the shorter the contact time and the wider the frequency range of the response. Nevertheless, apart its compactness, Hertz's theory remains a crude representation of the real interaction dynamics. Strictly speaking, it cannot be readily applied to dynamical problems for which the contact force evolves with time. But a quasi-static approximation, as considered here, is a plausible pragmatic approach.

Denoting $z(t)$ and $y^R(\mathbf{r}, t)$ the normal motion of the clapper and the bell respectively, the bell/clapper interaction force at $\mathbf{r}=\mathbf{r}_0$ is given by:

$$\begin{cases} f_c(t) = -K_c|z(t) - y^R(\mathbf{r}_0, t)|^b & \text{if} \quad z(t) < y^R(\mathbf{r}_0, t) \\ f_c(t) = 0 & \text{if} \quad z(t) \geq y^R(\mathbf{r}_0, t) \end{cases} \quad (14)$$

In a modal description, when the clapper and the bell are in contact, Eq. (14) becomes

$$f_c(t) = -K_c|z(t) - y^R(\mathbf{r}_0, t)|^b = -K_c\left|z(t) - \sum_{n=1}^{N} q_n(t)\varphi_n^R(\mathbf{r}_0)\right|^b \quad (15)$$

which highlights that all bell modes become coupled through the contact nonlinearity. This redistributes the impact energy across all the modes during the contact duration, even those which were not excited initially.

Equations (7), (8), (10), (11) and (14) represent the essential physics of the bell dynamics and surely includes enough details to obtain satisfactory simulations. A refined modelling of the contact force, as proposed in [30], could have been implemented but does not appear necessary for the purpose of this paper.

5. Dynamical simulations of the Coruche bell

5.1. Time-step integration

There are many time-step integration algorithms which might compute the time-domain responses of the bell to a given clapper excitation from the computational model. In our implementation, we used the explicit velocity-Verlet integration scheme [31] which is known to provide accurate results in the context of vibro-impacting systems [32,33]. Basically, the dynamical quantities, i.e., position, velocity and acceleration at time $t_{n+1} = t_n + \Delta t$ are computed from the corresponding quantities and the contact force at time t_n using an adequate time step Δt. Here, the main difficulty comes from the short duration of the contact time which requires a small Δt for stable and accurate computations. Indeed, contact time typically falls in the range 0.5–1.0 ms [30] and a large number of modes up to 5000 Hz are expected to be excited.

In order to achieve accurate simulations, a modal basis comprising $N = 110$ modes and covering the frequency range from 0 to 10000 Hz was used, and since the numerical efficiency was not an issue here, time-domain responses were sampled using a safe time step of 4.10^{-6} s. As already mentioned, modal damping values were estimated from a fitting procedure achieved on data pertaining to a modern bell, resulting in values ranging from 0.0002 to 0.001. Initially, at the beginning of contact, the bell and clapper are at rest and the clapper is subjected to a normal velocity $-\nu_0$. The impact velocity is $|\nu_0| = 0.1$ m/s and a value $b = 1.5$ is used for the Hertz law. Sound files were generated at the usual sampling frequency $f_s = 44{,}100$ Hz.

5.2. Time-domain numerical simulations

As for any percussion instrument, the quality of the sound radiated by a bell depends as much on the bell vibrational properties as on the way the clapper strikes the bell. Besides the vibrational modal parameters, three important factors influence the overall spectrum of bell sounds:

- the striking location which determines the contribution of every vibrational mode in the overall vibration;
- the dynamics of the contact which affects the distribution of the vibrational energy within the spectrum;
- the actual mechanical properties of the contacting bodies which controls the contact time.

To explore the acoustics of the bell from Coruche, extensive computations were therefore performed for different assumptions concerning the clapper mass, radius and material as well as the strike location and as a result, a range of plausible synthetic sounds were generated, evoking the musical qualities of this medieval bell.

As a first example, Fig. 7 shows the time-history and corresponding spectrogram of the bell velocity for a typical stroke, when the bell is impacted near the rim, by a clapper of mass $m_c = 0.2$ Kg and radius $r_c = 0.01$ m. As expected, the overall bell vibration decays slowly, with an evident beating pulsation due to the small asymmetries of the bell structure (section 4.2). It is also apparent that most of the vibrational energy rapidly lays in the first lower modes as the amplitudes of the higher-order modes die away with fast different decay rates. Musically, when struck, the bell produces a rich metallic sound which quickly gives way to a mellow sound dominated by its first partial, the hum tone. A typical sound for the Coruche bell is illustrated in the audio example 1.

A significant feature of bells which has been crucial regarding the evolution of their tuning [34] is the possible selective excitation of every partial when striking a bell at some specific location along the vertical direction. Indeed, striking the bell at different locations distributes the clapper energy into different bell modes during the bell/clapper interaction, and this leads to sounds with very different distributions of energy along the frequency. In Fig. 8, this situation is illustrated by simulating the bell response to successive similar impacts located at heights $h_c = 0.2h$, $0.4h$, $0.6h$, $0.8h$ and $0.95h$ with $h = 21$ cm being the bell height, every simulation being 10 s long. The velocity time responses of the bell are clearly different, with a net decrease of the amplitude as the excitation approaches the bell's head. Also, the corresponding spectrogram emphasizes the changes in the modal amplitudes of the vibrating modes as the impact location varies. As the excitation moves away from the rim, higher-order modes become more excited while the amplitudes of the lower modes decrease, leading to a gradual change of the timbral characteristics of the bell, hearing a rich and well-balanced sound first and finally, a thin sound, particularly rich in high frequencies and rather poor as far as the lower modes are concerned. This is illustrated in the audio example 2.

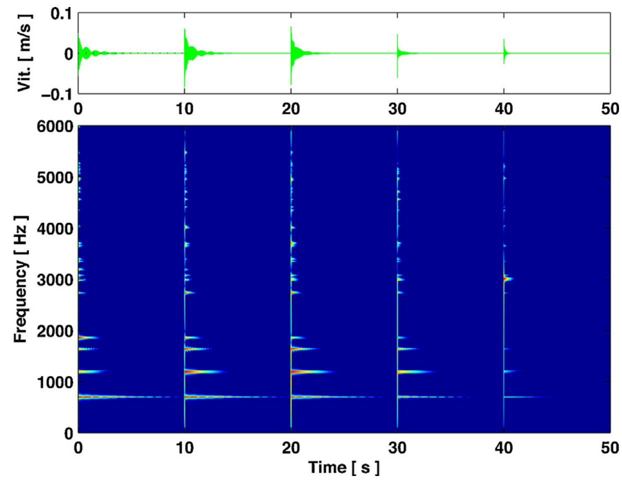

Fig. 8. Computed vibratory responses of the Coruche bell velocity at the impact location as a function of the impact location along the vertical direction, namely height $h_c = 0.2h$, $0.4h$, $0.6h$, $0.8h$ and $0.95h$. $m_c = 0.2$ Kg, $r_c = 0.01$ m, $\nu_0 = 0.1$ m/s.

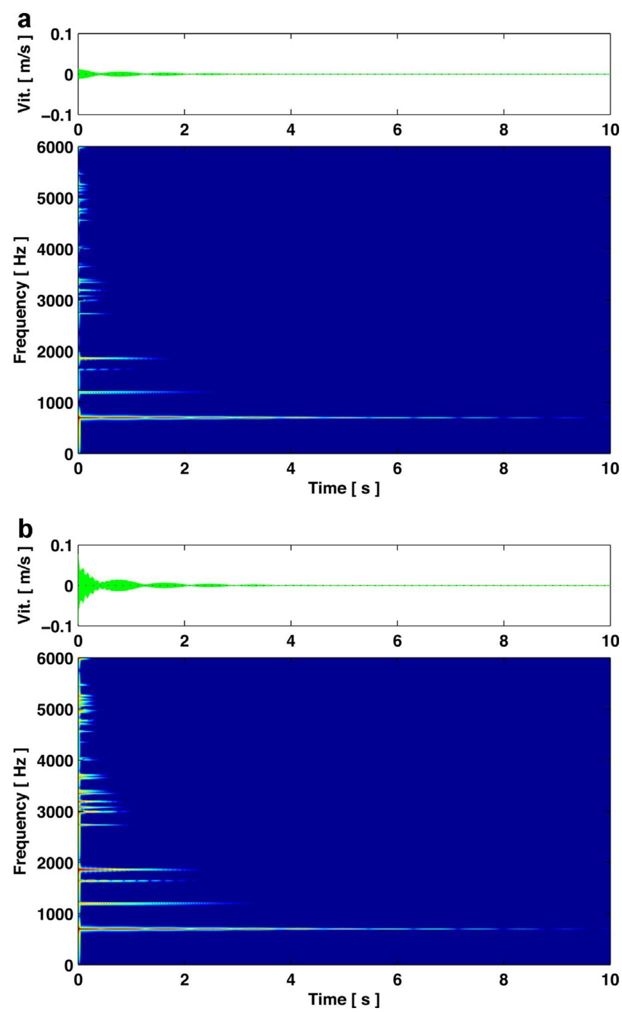

Fig. 9. Computed vibratory responses as a function of the contact stiffness. (a) $K_c = 10^6$ N/m$^{1.5}$, $h_c = 0.0475$ m, $m_c = 0.2$ Kg, $\nu_0 = 0.1$ m/s; (b) $K_c = 10^9$ N/m$^{1.5}$, $h_c = 0.0475$ m, $m_c = 0.2$ Kg, $\nu_0 = 0.1$ m/s.

In Fig. 9, we display the simulation results obtained by striking the bell with clappers of different stiffnesses. The case of softer clapper and harder clapper are considered by testing contact stiffnesses K_c of about 10^6 N/m$^{1.5}$ and 10^9 N/m$^{1.5}$, see Eq. (14). As expected, a

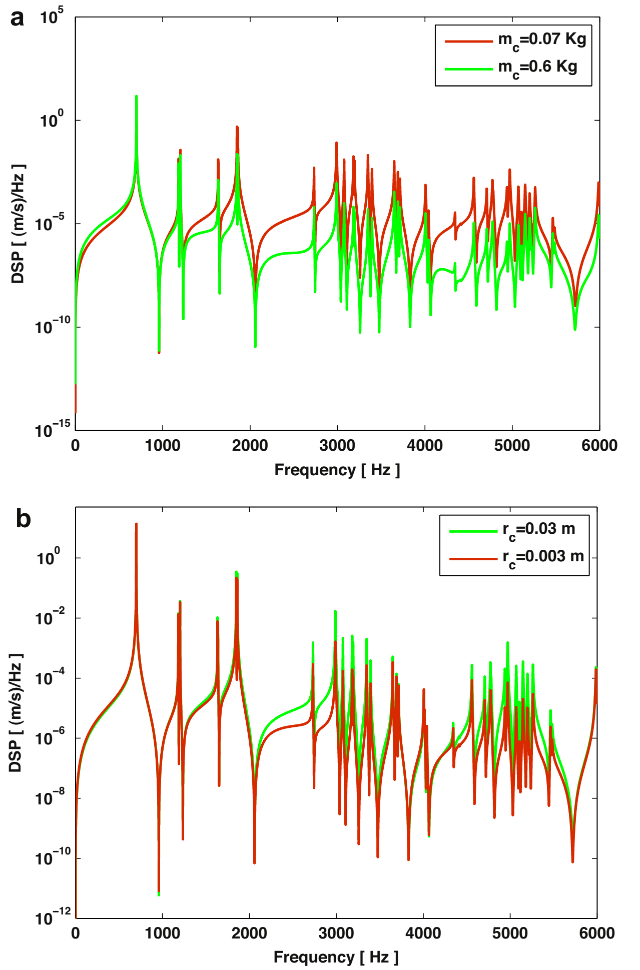

Fig. 10. Computed power spectrum of the bell velocity at the contact point as a function of the (a) clapper mass $r_c = 0.01$ m, $v_0 = 0.1$ m/s and (b) clapper ball radius $m_c = 0.2$ Kg, $v_0 = 0.1$ m/s.

hard clapper excites more efficiently a larger number of modes than a soft one, resulting in a brighter metallic sound (audio example 3). On the contrary, using a soft clapper results in a gentle sound, favoring few low-order modes (audio example 4).

Finally, Fig. 10 displays the results of parametric computations performed by modifying two parameters which directly control the stiffness of the contact. Fig. 10a illustrates the influence of the clapper mass, while Fig. 10b pertains to the case of varying the clapper ball radius. Spectra of the bell velocity at the contact location normalized to their energy are presented to emphasize the difference in their frequency contents. Similar to what is observed in impact tests, increasing the clapper mass increases the contact time, restricting the transfer of vibrational energy to the higher modes, and strongly excites the lower ones. Furthermore, enlarging the clapper radius significantly increases the number of modes excited. Perceptually, these differences in bell motion characteristics are accompanied by audible changes in brightness of the sound radiated. Both effects are illustrated in the audio examples 5 and 6 for the clapper mass, and 7 and 8 for the clapper radius.

6. Conclusions

In this paper, we assert the tuning properties of a damaged 13th century bell and resurrect its sounding qualities by combining experimental and numerical techniques from materials science and music acoustics. A modal computation of the bell vibrational properties was performed by the Finite Element Method and results were examined in the light of musical concerns. Modal computations were based on a 3D Finite Element model of the actual bell, reconstructed from geometrical data obtained by non-destructive measurements, and used calculated bell bronze mechanical properties, as Young modulus and density, based on the actual elemental and microstructural characteristics of the bronze, obtained by energy dispersive X-ray fluorescence spectrometry and optical and scanning electron microscopies. Time-domain simulations of the Coruche bell impacted by a clapper were performed by combining a physical modal-based model for a clapper-excited bell including the modal parameters identified by FEM computations. In particular, parametric simulations of the bell vibratory response were performed for different assumptions concerning the unknown bell clapper and impact characteristics, as well as for various strike locations, therefore evoking the unknown sounding qualities of this historical bell. Our results show that the Coruche bell has three partials reasonably in tune, which would give an identifiable musical pitch when struck.

By focusing on the vibrational aspects, the present approach is very adequate to understand much of the musical qualities of this medieval bell. The sound radiation represents a great deal of physics and more work is needed to compute the pressure field radiated by the bell. Such delicate and important aspects should be addressed by the authors. Also, a particular study concerning the Young modulus of delta phase in bronzes, essential for the development of the studies on historical bronze properties, should be performed. For cultural heritage concerns, the proposed methodology is straightforward and can be applied to other musical instruments which are no longer in condition to be played, either because they are broken, have missing parts or because of other particular conservations issues.

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Appendix A. Supplementary data

Supplementary data associated with this article can be found, in the online version, at http://dx.doi.org/10.1016/j.culher.2015.09.007.

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