NOVA SCHOOL OF SCIENCE & TECHNOLOGY
Department of Mathematics

Acesso ao Ensino Superior de Estudantes Internacionais
Math test – June 12, 2023
Duration: 2 hours

The candidate, when solving this test, undertakes not to resort to any type of consultation. No books, notes, calculators, computer and internet help, etc..
This test consists of 5 groups of questions and has a total score of 20 points.
Justify all your answers, showing all the steps, clearly indicating your reasoning and present the calculations made for full credit.
Each group of questions must be solved on separate sheets.

Points

[2,0] **1.** A JUNE12 code consists of a sequence of four letters where each letter belongs to the set $\{\mathtt{A}, \mathtt{B}, \mathtt{C}, \mathtt{D}, \mathtt{E}, \mathtt{F}\}$. Calculate the number of JUNE12 codes in which there are no repeating letters. (For example, `DCAF`, `BACE` and `FABD` are JUNE12 codes in the requested conditions.)

2. Determine the set of solutions of each of the following inequalities:

[2,0] (a) $\left(\frac{1}{4}\right)^{5-\frac{1}{2}x^2} \leq 2^{3x}$.

[2,0] (b) $\log_{\frac{1}{2}}\left(2^{-x}-2\right) \geq -1$.

[2,0] (c) $x - \frac{2x+10}{x-1} \leq 0$.

3. Compute the following limits of successions:

[2,0] (a) $\lim_{n\to+\infty} \frac{3^{n-5}-5^{n+4}}{3^{n+7}+5^{n+2}}$.

[2,0] (b) $\lim_{n\to+\infty} \left(\frac{4n-1}{4n-5}\right)^{4n-9}$.

4. Consider the function f, real of the variable real, defined by $f(x) = \begin{cases} \sqrt{5-|x^2-1|}, & x \leq 0 \\ \frac{\sin^2(x)}{1-\cos(x)}, & x > 0 \end{cases}$.

[2,0] (a) Determine the domain of the function f.

[2,0] (b) Study the continuity of the function f at $x = 0$.

[2,0] (c) Find the equation of the line tangent to the graph of f at the abscissa point $a = \frac{\pi}{2}$. (Give the equation in the form $y = mx + b$.)

[2,0] **5.** Prove the following trigonometric equality: $\frac{1+\cos(-x)}{x^2} = \frac{\sin^2(x)}{x^2 - x^2\cos(x)}$.

End of the test

(Generally useful formulas on the back of this sheet)

Generally useful formulas

- $\lim \left(1+\frac{1}{n}\right)^n = e \quad (n \in \mathbb{N})$
- $\lim_{x\to 0} \frac{e^x - 1}{x} = 1$
- $\lim_{x\to +\infty} \frac{e^x}{x^p} = +\infty \quad (p \in \mathbb{R})$
- $\lim_{x\to 0} \frac{\operatorname{sen} x}{x} = 1$
- $\operatorname{sen}(\alpha+\beta) = \operatorname{sen}\alpha\cos\beta + \operatorname{sen}\beta\cos\alpha$
- $\cos(\alpha+\beta) = \cos\alpha\cos\beta - \operatorname{sen}\alpha\operatorname{sen}\beta$
- Sum of the first n terms of a progression (u_n) with ratio r:

  Arithmetic progression: $\frac{u_1 + u_n}{2} \times n$

  Geometric progression: $u_1 \times \frac{1-r^n}{1-r}$

- Derivation rules:
  $(u+v)' = u' + v'$; $\quad (uv)' = u'v + uv'$; $\quad \left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$; $\quad (u^n)' = nu^{n-1}u' \quad (n \in \mathbb{R})$;
  $(\operatorname{sen} u)' = u'\cos u$; $\quad (\cos u)' = -u'\operatorname{sen} u$; $\quad (\operatorname{tg} u)' = \frac{u'}{\cos^2 u}$;
  $(e^u)' = u'e^u$; $\quad (a^u)' = u'a^u \ln a \quad (a \in \mathbb{R}^+ \setminus \{1\})$;
  $(\ln u)' = \frac{u'}{u}$; $\quad (\log_a u)' = \frac{u'}{u \ln a} \quad (a \in \mathbb{R}^+ \setminus \{1\})$

- $ax^2 + bx + c = 0$

  Quadratic formula: $x = \frac{-b \mp \sqrt{b^2 - 4ac}}{2a}$