The candidate, when solving this test, undertakes not to resort to any type of consultation. No books, notes, calculators, computer and internet help, etc..
This test consists of 4 groups of questions and has a total score of 20 points.
Justify all your answers, showing all the steps, clearly indicating your reasoning and present the calculations made for full credit.
Each group of questions must be solved on separate sheets.

Points

[2,0] **1.** Consider, in a plane β, two parallel straight lines r and s. Five distinct points are marked on the line r, and a certain number n of distinct points are marked on the line s. It is known that, with the points marked on the two lines, it is possible to define exactly 100 triangles. Determine the value of n.

2. Determine the set of solutions for each of the following inequalities:

[2,0] (a) $3^{\frac{x^2-4}{x^2+5}} < 1$.

[2,0] (b) $x^3 \log_2(2x) - x^3 \log_2(x+5) < 0$.

[2,0] (c) $\left|\dfrac{2x-5}{x-7}\right| \leq 10$.

3. Compute the following limits of successions:

[2,0] (a) $\lim\limits_{n\to+\infty} \left(\sqrt{n!} - \sqrt{(n+1)!}\right)$.

[2,0] (b) $\lim\limits_{n\to+\infty} \left[\left(1-\dfrac{1}{n}\right)^n \sqrt[n]{\dfrac{n+1}{n}}\right]$.

[2,0] (c) $\lim\limits_{n\to+\infty} \left(\dfrac{n^2-3}{n^2}\right)^n$.

4. Consider the function f, real of the variable real, defined by $f(x) = \begin{cases} e^x - 1, & x \geq 0 \\ \cos(x)\ln(x+1), & x < 0 \end{cases}$.

[2,0] (a) Determine the domain of the function f and study the continuity of f.

[2,0] (b) Prove that exists $a \in \left]-\dfrac{\pi}{4}, 1\right[$ such that $f(a) = 0$.

[2,0] (c) Justify that the function has a maximum and a minimum in the interval $[0,1]$. Indicate its values.

Fim

(Generally useful formulas on the back of this sheet)

Generally useful formulas

- $\lim_{n\to+\infty}\left(1+\frac{1}{n}\right)^n = e \quad (n \in \mathbb{N})$
- $\lim_{x\to 0}\frac{e^x - 1}{x} = 1$
- $\lim_{x\to+\infty}\frac{e^x}{x^p} = +\infty \quad (p \in \mathbb{R})$
- $\lim_{x\to 0}\frac{\sin x}{x} = 1$
- $\sin^2(\alpha) + \cos^2(\alpha) = 1$
- $\sin(\alpha+\beta) = \sin\alpha\cos\beta + \sin\beta\cos\alpha$
- $\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$
- Sum of the first n terms of a progression (u_n) with ratio r:

  Arithmetic progression: $\frac{u_1 + u_n}{2} \times n$

  Geometric progression: $u_1 \times \frac{1 - r^n}{1 - r}$
- Derivation rules:
  $(u+v)' = u' + v'$; $(uv)' = u'v + uv'$; $\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$; $(u^n)' = nu^{n-1}u' \quad (n \in \mathbb{R})$;
  $(\sin u)' = u'\cos u$; $(\cos u)' = -u'\sin u$; $(\operatorname{tg} u)' = \frac{u'}{\cos^2 u}$;
  $(e^u)' = u'e^u$; $(a^u)' = u'a^u \ln a \quad (a \in \mathbb{R}^+ \setminus \{1\})$;
  $(\ln u)' = \frac{u'}{u}$; $(\log_a u)' = \frac{u'}{u \ln a} \quad (a \in \mathbb{R}^+ \setminus \{1\})$
- $ax^2 + bx + c = 0$

  Quadratic formula: $x = \frac{-b \mp \sqrt{b^2 - 4ac}}{2a}$