NOVA SCHOOL OF SCIENCE & TECHNOLOGY
Departamento de Matemática

Acesso ao Ensino Superior de Estudantes Internacionais
Math test – May 23, 2025
Duration: 2 hours

The candidate, when solving this test, undertakes not to resort to any type of consultation. No books, notes, calculators, computer and internet help, etc..
This test consists of 4 groups of questions and has a total score of 20 points.
Justify all your answers, showing all the steps, clearly indicating your reasoning and present the calculations made for full credit.
Each group of questions must be solved on separate sheets.

Points

[2,0] **1.** (a) On a train, there is a group of eight friends and in this group are three siblings. In how many different ways can these eight friends get off the train so that the three siblings get off consecutively?

[2,0] (b) A MAY23 code consists of a sequence of six letters where each letter belongs to the set $\{\mathtt{A}, \mathtt{B}, \mathtt{C}, \mathtt{D}\}$. Calculate the number of MAY23 codes that simultaneously use the four letters `A`, `B`, `C`, and `D`.
(For example, `DCAABB`, `DCACBA`, and `DBADCC` are MAY23 codes under the requested conditions.)

2. Determine the set of solutions for each of the following inequalities:

[2,0] (a) $(9-x^2)3^x < 0$.

[2,0] (b) $x \log_5(x^2-1) > x$.

[2,0] (c) $\left|\dfrac{x-2}{x+4}\right| > 1$.

3. Compute the following limits of successions:

[2,0] (a) $\lim\limits_{n\to+\infty} \left(\dfrac{3}{n^3+7} \times \sqrt{n-5}\right)$.

[2,0] (b) $\lim\limits_{n\to+\infty} \left(\dfrac{3^n + (-5)^{n+1}}{4^{n+2} - 3^n}\right)$.

[2,0] (c) $\lim\limits_{n\to+\infty} \left(\dfrac{n^3-5}{n^3}\right)^{n^2-5}$.

4. Consider the function f, real of the variable real, defined by $f(x) = \begin{cases} x+3a, & x \leq 2 \\ \dfrac{x(x-2)}{x^2-5x+6}, & x > 2 \end{cases}$.

[2,0] (a) Determine the domain of the function f and calculate the value of parameter a such that the function f will be continuous at $x = 2$.

[2,0] (b) Find the equation of the line tangent to the graph of f at the abscissa point $b = 6$.
(Give the equation in the form $y = mx + b$.)

End of the test

(Generally useful formulas on the back of this sheet)

Generally useful formulas

- $\lim_{n\to+\infty}\left(1+\frac{1}{n}\right)^n = e \quad (n\in\mathbb{N})$
- $\lim_{x\to 0}\frac{e^x-1}{x} = 1$
- $\lim_{x\to+\infty}\frac{e^x}{x^p} = +\infty \quad (p\in\mathbb{R})$
- $\lim_{x\to 0}\frac{\sin x}{x} = 1$
- $\sin^2(\alpha)+\cos^2(\alpha) = 1$
- $\sin(\alpha+\beta) = \sin\alpha\cos\beta+\sin\beta\cos\alpha$
- $\cos(\alpha+\beta) = \cos\alpha\cos\beta-\sin\alpha\sin\beta$
- Sum of the first n terms of a progression (u_n) with ratio r:

  Arithmetic progression: $\frac{u_1+u_n}{2}\times n$

  Geometric progression: $u_1\times\frac{1-r^n}{1-r}$

- Derivation rules:
  $(u+v)' = u'+v'$; $(uv)' = u'v+uv'$; $\left(\frac{u}{v}\right)' = \frac{u'v-uv'}{v^2}$; $(u^n)' = nu^{n-1}u' \quad (n\in\mathbb{R})$;
  $(\sin u)' = u'\cos u$; $(\cos u)' = -u'\sin u$; $(\operatorname{tg} u)' = \frac{u'}{\cos^2 u}$;
  $(e^u)' = u'e^u$; $(a^u)' = u'a^u\ln a \quad (a\in\mathbb{R}^+\setminus\{1\})$;
  $(\ln u)' = \frac{u'}{u}$; $(\log_a u)' = \frac{u'}{u\ln a} \quad (a\in\mathbb{R}^+\setminus\{1\})$
- $ax^2+bx+c=0$

  Quadratic formula: $x = \frac{-b\mp\sqrt{b^2-4ac}}{2a}$